Standard 2: Operations and Algebraic Thinking

	K	1	2	3	4	5	6	7	8	9	10	11	12
							Pre-Algebra	Algebra I	Geometry	Algebra II	Fourth Course		
Benchmark 1	Add and subtract within 5.	Add and subtract within 20.	Add and subtract within 20.	Multiply and divide within 100.	Gain familiarity with factors and multiples.	Gain familiarity with factors and multiples.	Compute fluently with multi-digit numbers and find common factors and multiples.						
Operations	Fluently add and subtract within 5.	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).	Fluently add and subtract within 20 using mental strategies.	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division or properties of operations.	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	Find all factor pairs for whole numbers in the range of 1-100. Determine whether a given whole number in the range of 1-100 is prime or composite. Find the prime factorization of numbers.	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12.						
		Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.	Know from memory all sums of two one-digit numbers.	Know from memory all products of two one-digit numbers.									
Benchmark 2		Work with addition and subtraction equations.	Work with equal groups of objects to gain foundations for multiplication.	Solve problems involving the four operations, and identify and explain patterns in arithmetic.	Generate and analyze patterns.	Analyze patterns and relationships.	Represent and analyze quantitative relationships between dependent and independent variables.						
Solving Equations		Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.	Determine whether a group of objects (up to 20) has an odd or even number of members.	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.						
		Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.									
Benchmark 3	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving addition and subtraction.	Represent and solve problems involving multiplication and division.	Use the four operations with whole numbers to solve problems.	Write and interpret numerical expressions.	Apply and extend previous understandings of arithmetic to algebraic expressions.	Use properties of operations to generate equivalent expressions.	Interpret the structure of expressions. (Linear, exponential, quadratic)	Interpret the structure of expressions. (Polynomial and Rational)			
Seeing Structure in Expressions	Represent addition and subtraction with objects, fingers, mental images, drawings1, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.	Use addition and subtraction within 100 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.	Interpret products of whole numbers.	Interpret a multiplication equation as a comparison.	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	Write and evaluate numerical expressions involving whole-number exponents.	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	Interpret expressions that represent a quantity in terms, factors, and coefficients.	Interpret expressions that represent a quantity in terms of its context (polynomial and rational).			
	Solve addition and subtraction word problems, and add and subtract within 10.	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.		Interpret whole-number quotients of whole numbers.	Multiply or divide to solve word problems involving multiplicative comparison.	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.	Write, read, and evaluate expressions in which letters stand for numbers.	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.	Use the structure of an expression to identify ways to rewrite it.	Use the structure of an expression to identify ways to rewrite it (polynomial and rational).			
	Decompose numbers less than or equal to 10 into pairs in more than one way.			Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.								

	For any number from 1 to 9, find the number that makes 10 when added to the given number.		Determine the unknown whole number in a multiplication or division equation relating three whole numbers.										
Benchmark 4	**Understand and apply properties of operations and the relationship between addition and subtraction.**		**Understand properties of multiplication and the relationship between multiplication and division.**			**Apply properties of operations**	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations**	**Write expressions in equivalent forms to solve problems. (Quadratic and exponential)**		**Write expressions in equivalent forms to solve problems. (Quadratic and exponential)**			
Expressions and Equations	Apply properties of operations as strategies to add and subtract.		Apply properties of operations as strategies to multiply and divide.			Apply the properties of operations to generate equivalent expressions.	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*		Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.*			
	Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8. Add and subtract within 20.		Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.			Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.						
Benchmark 5								**Perform arithmetic operations on polynomials. (Linear and quadratic)**		**Perform arithmetic operations on polynomials. (Beyond Quadratic)**			
Arithmetic with Polynomials and Rational Expressions								Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.		Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.			
Benchmark 5 (Part 2)										**Understand the relationship between zeros and factors of polynomials.**			
										Know and apply the Remainder Theorem:			
										Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.			
Benchmark 5 (Part 3)										**Use polynomial identities to solve problems.**			
										Prove polynomial identities and use them to describe numerical relationships. triples.			
										(+) Know and apply the Binomial Theorem for the expansion of $(x + y)n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.1			
Benchmark 5 (Part 4)										**Rewrite rational expressions.**			
										Rewrite simple rational expressions in different forms.			
										(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.			

Benchmark 6									**Create equations that describe numbers or relationships (Linear, quadratic, and exponential) (integer inputs only) for A.CED.3, linear only)**		**Create equations that describe numbers or relationships (equations using all available types of expressions, including simple and root functions.)**	
Creating Equations									Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.		Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	
									Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.		Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	
									Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.		Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.	
									Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.		Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R.	
Benchmark 7 (Part 1)							**Reason about and solve one-variable equations and inequalities.**	**Understand the connections between proportional relationships, lines, and linear equations.**	**Understand solving equations as a process of reasoning and explain the reasoning (Master linear, learn as general principle)**		**Understand solving equations as a process of reasoning and explain the reasoning (Simple radical and rational)**	
Reasoning with Equations and Inequalities							Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true?	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.		Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	
							Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number.	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.				
							Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.					
							Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem.					
Benchmark 7 (Part 2)								**Analyze and solve linear equations and pairs of simultaneous linear equations**	**Analyze and solve linear equations and pairs of simultaneous linear equations**			
								Solve linear equations in one variable.	Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.			

								Analyze and solve pairs of simultaneous linear equations.	Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.				
									Solve real-world and mathematical problems leading to two linear equations in two variables.				
Benchmark 7 (Part 3)									**Solve systems of equations (Linear-linear, and linear-quadratic)**			**Solve systems of equations.**	
									Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.			(+) Represent a system of linear equations as a single matrix equation in a vector variable.	
									Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.			(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	
									Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.				
Benchmark 7 (Part 4)									**Represent and solve equations and inequalities graphically (Linear and exponential; learn as general principle)**			**Represent and solve equations and inequalities graphically (Combine polynomial, rational, radical, absolute value, and exponential functions)**	
									Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).				
									Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the function , make table of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.			Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	
									Graph the solutions to a linear inequality in two variables as a half- plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes				